CS 1502 Exam II

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**Instructions:** This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

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**Question 1 (25 points)** Translate the following English sentences into first-order sentences, where the first-order language has the following:

- predicate symbol $L(x, y, z)$ to be interpreted as “$x$ likes $y$ at time $z$”.
- constant symbol $m$ to be interpreted as “I” or “me”.
- function symbol $f(x)$ to be interpreted as “$x$’s best friend”.

a) I always like people who like my best friend.
b) Sometimes I don’t like myself.
c) I always like people whose best friend likes me.
d) I never liked my best friend’s best friend.
e) I’m the only person who ever liked me.
Question 2 (25 points)

a) State the Universal Introduction (∀ Into) rule of inference for the Fitch system.

b) State the ∃ Elim rule of inference for the Fitch system.

c) Construct a formal Fitch style proof using only the primitive rules of inference (i.e., no Taut Con, Ana Con, or FO Con uses) showing that ∀xP(x) ∧ ∀xQ(x) is a first-order consequence of ∀x(P(x) ∧ Q(x)).
Question 3 (25 points) Using the Resolution Method show that the sentence $\forall x \exists y U(y, x)$ is a first-order consequence of the sentences $\forall w H(w)$ and $\forall x[H(x) \rightarrow \exists y U(y, x)]$. 
Question 4 (25 points)

a) Give the definition of a first-order structure $M$ for a first-order language $L$.

b) Show that $\exists x P(x) \lor \exists x Q(x)$ is a first-order consequence of $\exists x (P(x) \lor Q(x))$ by showing for any model $M$ that if $M \models \exists x (P(x) \lor Q(x))$ then $M \models \exists x P(x) \lor \exists x Q(x)$. 