CS 1502 Exam II

Robert Daley

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Instructions: This is a closed book, note and neighbor exam! You must show all work in the space provided on this test.

Name: _______________________

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Question 1 (25 points) Translate the following English sentences into first-order sentences, where the first-order language with equality has the following:

- predicate symbol L(x, y, z) to be interpreted as “x likes y at time z”.
- predicate symbol H(x, z) to be interpreted as “x is happy at time z”.
- constant symbol m to be interpreted as “I” or “me”.
- function symbol f(x) to be interpreted as “x’s best friend”.
- function symbol p(x) to be interpreted as “x’s pet”.

a) I’m always happy whenever someone likes my pet.

b) My best friend never fails to like my pet.

c) My pet is my best friend.

d) My best friend’s pet doesn’t like unhappy people.

e) Sometimes I’m unhappy and don’t like anybody.
Question 2 (25 points)

a) State the General Conditional Proof (∀ Into) rule of inference for the Fitch system.

b) State the ∃ Intro rule of inference for the Fitch system.

c) Construct a formal Fitch style proof using only the primitive rules of inference (i.e., no Taut Con, Ana Con, or FO Con uses) showing that ∃xP(x) ∨ ∃xQ(x) is a first-order consequence of ∃x(P(x) ∨ Q(x)).
Question 3 (25 points) Using the Resolution Method show that the sentence $\exists z \forall w (L(w, x) \land R(z))$ is a first-order consequence of the sentences $\exists x H(x)$, $\exists x R(x)$ and $\forall x \exists z [H(x) \rightarrow \forall y L(z, y)]$. 
Question 4 (25 points)

a) Give the definition of a first-order structure $\mathcal{M}$ for a first-order language $\mathcal{L}$.

b) Show that $\neg\forall x P(x)$ and $\exists x \neg P(x)$ are first-order equivalent by showing for any model $\mathcal{M}$ that $\mathcal{M} \models \neg\forall x P(x)$ if and only if $\mathcal{M} \models \exists x \neg P(x)$. 