CS 1502 Exam II

Robert Daley

24 March 2005

Instructions: This is a closed book, note and neighbor exam! You must show all work in the space provided on this test.

Name: ______________________

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Question 1 (25 points) Translate the following English sentences into first-order sentences, where the first-order language has the following:

- predicate symbol L(x, y) to be interpreted as “x likes y”.
- predicate symbol M(x) to be interpreted as “x is a CS major”.
- predicate symbol S(x) to be interpreted as “x is a Pitt student”.
- predicate symbol C(x) to be interpreted as “x is a CS course”.
- constant symbol m to be interpreted as “I” or “me”.
- constant symbol c to be interpreted as “CS1502”.
- function symbol r(x) to be interpreted as “x’s roommate”.

a) All Pitt students like CS1502.
b) Some Pitt students are CS majors.
c) Some Pitt students who are not CS majors don’t like CS1502.
d) My roommate likes all the courses that I don’t like.
e) Some CS majors like all of their courses.
Question 2 (25 points)

a) State the $\forall$ Elim rule of inference for the Fitch system.

b) State the $\exists$ Elim rule of inference for the Fitch system.

c) Construct a formal Fitch style proof using only the primitive rules of inference (i.e., no Taut Con, Ana Con, or FO Con uses) showing that $\neg\exists x \neg P(x)$ is a first-order consequence of $\forall x P(x)$. 

**Question 3 (25 points)** Using the Resolution Method show that \( \exists z [R(z)] \) is a first-order consequence of the sentences \( \exists w L(w), \forall x \exists y [C(x) \rightarrow R(y)] \) and \( \forall y [L(y) \rightarrow C(y)] \).
Question 4 (25 points)

a) Give the definition of a first-order structure $\mathcal{M}$ for a first-order language $\mathcal{L}$.

b) Show that $\forall x \exists y R(x, y)$ is a first-order consequence of $\exists y \forall x R(x, y)$, by showing for any model $\mathcal{M}$ that if $\mathcal{M} \models \exists y \forall x R(x, y)$, then it follows that $\mathcal{M} \models \forall x \exists y R(x, y)$.