

CS 1511 Exam I

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2 October 2002

Instructions: This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

Name: _____

Question	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Question 1 (25 points)

- a) Construct (the state transition diagram for) a Turing Machine that is a decider for the language

$$\{w \in \{a, b\}^* : w \text{ has equal a's and b's } \}.$$

[Be sure to include all transitions.]

- b) Give a brief description of how your TM works.
- c) Show (by giving the exact sequence of TM configurations) how your TM works on the input string *abbaba*.

Question 2 (25 points)

a) Construct a Context-Free Grammar that generates the language

$$\{w\#x : x \text{ is a substring of } w^{\mathcal{R}}, \text{ for } w, x \in \{a, b\}^*\}.$$

b) Give a leftmost derivation for $abab\#ab$ with respect to your grammar.

c) Give the derivation tree for the string $abab\#ab$ with respect to your grammar.

Question 3 (25 points)

- a) Construct (the state transition diagram for) a Push-Down Automaton that recognizes the language of all strings of properly nested parentheses. For example, “()”, “()()” and “(()())” are properly nested, but “)()” and “(())” are not.
[*Be sure to include all transitions.*]
- b) Give a brief English description of how your PDA works.
- c) Show (by means of a detailed step by step trace which includes the state of the input, PDA state, and stack) how it works on the input “(()())”.

Question 4 (25 points)

a) State the Pumping Lemma for:

- i) Regular Languages
- ii) Context-Free Languages

[Be sure to include all required quantifiers and components of the statements.]

b) Consider the following language

$$L = \{a^m b^n c^n : m, n \geq 0\}$$

- i) Is L context-free?
- ii) If L is **not** context-free, then prove this by using the Pumping Lemma for Context-Free Languages. **Otherwise**, prove it is not regular by using the Pumping Lemma for Regular Languages.