CS 1511 Exam II

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Instructions: This is a closed book, note and neighbor exam! You must show all work in the space provided on this test.

Name: ____________________________

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Question 1 (20 points)

a) Prove that the language $E_{CFG}$ is decidable. Be sure to give complete details of the proof in addition to the details of your construction.

b) Is your decider an LBA? Explain.
Question 2 (20 points) Using a diagonalization argument prove that the set \( \{ f \mid f : \Sigma^* \rightarrow \Sigma^* \} \) is uncountable.
Question 3 (20 points)

a) Give the definition of a computable function $f : \Sigma^* \rightarrow \Sigma^*$.

b) Give the definition of $A \leq_m B$ for languages $A$ and $B$.

c) Prove that $A_{TM} \not\leq_m E_{TM}$. 
Question 4 (20 points) Prove that $E_{TM}$ is not decidable. Be sure to give complete details of the proof in addition to the details of your construction.
Question 5 (20 points)

a) State the fixed-point version of the Recursion Theorem.

b) Let \( t : \Sigma^* \longrightarrow \Sigma^* \) be a transformation that reverses every transition in the state transition diagram of a Turing Machine \( \text{except} \) transitions to \( q_{\text{accept}} \) and \( q_{\text{reject}} \). Give an example of a \textbf{non-empty} decidable language \( L \) over \( \{a, b\}^* \) and a Turing Machine decider \( M \) for \( L \) that is a fixed-point for the transformation \( t \). Explain in detail why \( M \)'s behavior is immune to this transformation.
(Note: consider \( M \) to output “yes” if \( w \in L \) and “no” if \( w \not\in L \).)
(Hint: consider a very simple Turing Machine that behaves like a DFA)