CS 1511 Exam III

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Instructions: This is a closed book, note and neighbor exam! You must show all work in the space provided on this test.

Name: ____________________

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Question 1 (25 points)

a) Give the definition of the $TQBF$ problem.

b) Prove that $TQBF \in PSPACE$.
   Be sure to include correctness and complexity bounds in your proof.
Question 2 (25 points)

a) Give the definition of the PATH problem.

b) Prove that PATH ∈ P.
   Be sure to include correctness and complexity bounds in your proof.
**Question 3 (25 points)** State whether the following statements are TRUE or FALSE or UNKNOWN and **Explain** your answer.

a) $TQBF$ is NP-hard.

b) If $C$ is NP-complete and $A \leq_p C$ and $A \in NP$, then $A$ is NP-complete.

c) If $CLIQUE$ is NP-complete, then $P = NP$.

d) $E_{CFG}$ is NP-complete.

e) $3SAT$ is PSPACE-complete.
Question 4 (25 points)

a) Give the definition of the VERTEXCOVER problem.

b) Prove that VERTEXCOVER is a member of NP by constructing
   i) a polynomial time verifier for VERTEXCOVER, and
   ii) a polynomial time non-deterministic Turing machine that decides VERTEXCOVER.

c) Illustrate the polynomial time reduction \(3SAT \leq_p VERTEXCOVER\) for the boolean formula

\[
(x \lor y \lor z) \land (\overline{x} \lor \overline{y} \lor \overline{z}) \land (\overline{x} \lor y \lor \overline{z})
\]

by constructing the corresponding graph, and, if satisfiable, indicating the corresponding vertex cover.