

# CS 1511 Exam III

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**Instructions:** This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

**Name:** \_\_\_\_\_

Question	Percent	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

**Question 1 (25 points)**

- a) Give the definition of the  $TQBF$  problem.
- b) Prove that  $TQBF \in PSPACE$ .  
Be sure to include correctness and complexity bounds in your proof.

**Question 2 (25 points)**

- a) Give the definition of the *PATH* problem.
- b) Prove that  $PATH \in P$ .  
Be sure to include correctness and complexity bounds in your proof.

**Question 3 (25 points)** State whether the following statements are TRUE or FALSE or UNKNOWN and **Explain** your answer.

- a)  $TQBF$  is NP-hard.
- b) If  $C$  is NP-complete and  $A \leq_p C$  and  $A \in NP$ , then  $A$  is NP-complete.
- c) If  $CLIQUE$  is NP-complete, then  $P = NP$ .
- d)  $E_{CFG}$  is NP-complete.
- e)  $3SAT$  is PSPACE-complete.

**Question 4 (25 points)**

- a) Give the definition of the *VERTEXCOVER* problem.
- b) Prove that *VERTEXCOVER* is a member of NP by constructing
  - i) a polynomial time verifier for *VERTEXCOVER*, and
  - ii) a polynomial time non-deterministic Turing machine that decides *VERTEXCOVER*.
- c) Illustrate the polynomial time reduction  $3SAT \leq_p VERTEXCOVER$  for the boolean formula

$$(x \vee y \vee z) \wedge (\bar{x} \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee \bar{z})$$

by constructing the corresponding graph, and, if satisfiable, indicating the corresponding vertex cover.