

# CS 1511 Exam I

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6 February 2003

**Instructions:** This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

**Name:** \_\_\_\_\_

Question	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

**Question 1 (25 points)**

- a) Construct (the state transition diagram for) a deterministic single-tape Turing Machine that is a decider for the language

$$L = \{a^n b^{2n} : n \geq 1\}$$

*[Be sure to include all transitions.]*

- b) Give a brief description of how your TM works.
- c) Show (by giving the exact sequence of TM configurations) how your TM works on the input string *aabbb*.

**Question 2 (25 points)**

- a) Construct a Context-Free Grammar that generates the language

$$L = \{a^m b^n : m = n \text{ or } m = 2n\}$$

- b) Give a leftmost derivation for  $aaaabb$  with respect to your grammar.
- c) Give the derivation tree for the string  $aaaabb$  with respect to your grammar.

**Question 3 (25 points)**

- a) Construct (the state transition diagram for) a Push-Down Automaton that recognizes the language

$$\{w\#x : x \text{ is a substring of } w^{\mathcal{R}}, \text{ for } w, x \in \{a, b\}^*\}.$$

*[Be sure to include all transitions.]*

- b) Give a brief English description of how your PDA works.
- c) Show (by means of a detailed step by step trace which includes the state of the input, PDA state, and stack) how it works on the input  $abab\#ab$ .

**Question 4 (25 points)**

a) State the Pumping Lemma for:

- i) Regular Languages
- ii) Context-Free Languages

*[Be sure to include all required quantifiers and components of the statements.]*

b) Consider the following language

$$L = \{a^{n!} : n \geq 0\}$$

- i) Is  $L$  context-free?
- ii) If  $L$  is **not** context-free, then prove this by using the Pumping Lemma for Context-Free Languages. **Otherwise**, prove it is not regular by using the Pumping Lemma for Regular Languages.