CS 1511 Exam III

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Instructions: This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

Name: _____

Question	Percent	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Question 1 (25 points) Consider the following two-person **Generalized Wall** game:

- The game is played on an $n \times n$ checkerboard, that initially has one square with a black checker on it and one with a red checker on it.
- Two players, Player 1 and Player 2, take turns making moves.
- Player 1 moves first and has black checkers, and Player 2 has red checkers.
- During their turn each player must place one of their checkers on an unoccupied square that is *adjacent* to a square with one of their of checkers.
- The first player who is unable to make a legal move loses.

Define

$GW = \{< n, b, r > \mid$ Player 1 has a winning strategy for the Generalized Wall game on an $n \times n$ checkerboard, b marks the location of the inital black checker, and r marks the location of the initial red checker $\}$

Prove that $GW \in PSPACE$.

Be sure to include correctness and complexity bounds in your proof.

Question 2 (25 points)

a) Give the definition of the $TRIANGLE$ problem.

b) Prove that $TRIANGLE \in P$.
Be sure to include correctness and complexity bounds in your proof.

Question 3 (25 points) Fill in the blanks with the following terms, where no term may be used **more than once** (any such occurrence will be marked WRONG).

a) _____ is NP-complete.

b) _____ is **not** decided by a deterministic polynomial space Turing machine.

c) _____ is PSPACE-complete.

d) _____ is decided by a deterministic polynomial time Turing machine.

e) _____ is decided by a deterministic polynomial space Turing machine.

- $\overline{E_{LBA}}$
- $\overline{E_{NFA}}$
- GG
- $HAMPATH$
- $PATH$

Question 4 (25 points)

a) Give the definition of the $SUBSETSUM$ problem.

b) Prove that $SUBSETSUM$ is a member of NP by constructing

   i) a polynomial time verifier for $SUBSETSUM$, and

   ii) a polynomial time non-deterministic Turing machine that decides $SUBSETSUM$.

c) Illustrate the polynomial time reduction $3SAT \leq_p SUBSETSUM$ for the boolean formula

$$(x \vee y \vee z) \wedge (\overline{x} \vee \overline{y} \vee \overline{z}) \wedge (\overline{x} \vee y \vee \overline{z})$$

   by constructing the corresponding graph, and, if satisfiable, indicating the corresponding subset.