CS 1511 Exam III

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Instructions: This is a closed book, note and neighbor exam! You must show all work in the space provided on this test.

Name: ____________________________

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**Question 1 (25 points)** Consider the following two-person Generalized Wall game:

- The game is played on an $n \times n$ checkerboard, that initially has one square with a black checker on it and one with a red checker on it.

- Two players, Player 1 and Player 2, take turns making moves.

- Player 1 moves first and has black checkers, and Player 2 has red checkers.

- During their turn each player must place one of their checkers on an unoccupied square that is adjacent to a square with one of their of checkers.

- The first player who is unable to make a legal move loses.

Define

$$GW = \{ < n, b, r > \mid \text{Player 1 has a winning strategy for the Generalized Wall game on an } n \times n \text{ checkerboard, } b \text{ marks the location of the initial black checker, and } r \text{ marks the location of the initial red checker} \}$$

Prove that $GW \in PSPACE$.

Be sure to include correctness and complexity bounds in your proof.
Question 2 (25 points)

a) Give the definition of the $TRIANGLE$ problem.

b) Prove that $TRIANGLE \in P$.
   
   Be sure to include correctness and complexity bounds in your proof.
Question 3 (25 points) Fill in the blanks with the following terms, where no term may be used more than once (any such occurrence will be marked WRONG).

a) \underline{GG} is NP-complete.

b) \underline{HAMPATH} is not decided by a deterministic polynomial space Turing machine.

c) \underline{PATH} is PSPACE-complete.

d) \underline{PATH} is decided by a deterministic polynomial time Turing machine.

e) \underline{ELBA} is decided by a deterministic polynomial space Turing machine.

- \underline{ELBA}
- \underline{ENFA}
- \underline{GG}
- \underline{HAMPATH}
- \underline{PATH}
Question 4 (25 points)

a) Give the definition of the \textit{SUBSETSUM} problem.

b) Prove that \textit{SUBSETSUM} is a member of NP by constructing
   i) a polynomial time verifier for \textit{SUBSETSUM}, and
   ii) a polynomial time non-deterministic Turing machine that decides \textit{SUBSETSUM}.

c) Illustrate the polynomial time reduction $3SAT \leq_p \textit{SUBSETSUM}$ for the boolean formula

\[ (x \lor y \lor z) \land (\overline{x} \lor \overline{y} \lor \overline{z}) \land (\overline{x} \lor y \lor \overline{z}) \]

by constructing the corresponding graph, and, if satisfiable, indicating the corresponding subset.