CS 1511 Exam II

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Instructions: This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

Name: _____

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Question 1 (20 points)

a) Prove that the language

$S = \{< M > \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w\}$

is decidable. Be sure to give complete details of the proof in addition to the details of your construction.

b) Is your decider an LBA? Explain.

Question 2 (20 points)

a) Using a *diagonalization argument* prove that for any alphabet Σ there are uncountably many languages over Σ^*.

b) Use part a) to prove that there must exist languages that are not Turing recognizable.

Question 3 (20 points)

a) Give the definition of $A \leq_m B$ for languages A and B.

b) Prove for every context free language L that $L \leq_m A_{CFG}$.

c) Prove that $E_{TM} \not\leq_m A_{TM}$.

Be sure to give complete details of the proofs in addition to the details of your constructions.

Question 4 (20 points) Prove that the language ALL_{TM} is not decidable. Be sure to give complete details of the proofs in addition to the details of your constructions.

Question 5 (20 points)

a) State the fixed-point version of the Recursion Theorem.

b) Let $t : \Sigma^* \longrightarrow \Sigma^*$ be a transformation that interchanges the tape symbols 0 and 1 in the state transition diagram of a Turing Machine with input alphabet $\{0, 1\}$. Give an example of a **non-trivial** decidable language L over $\{0, 1\}^*$ and a Turing Machine decider M for L that is a fixed-point for the transformation t. Explain in detail why M's behavior is immune to this transformation.
(Note: *non-trivial* L means that $L \neq \emptyset$ and $L \neq \{0, 1\}^*$.)
(Note: consider M to output "yes" if $w \in L$ and "no" if $w \notin L$.)
(Hint: consider a very simple Turing Machine that behaves like a DFA)