CS 1511 Exam III

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Instructions: This is a closed book, note and neighbor exam! You must **show all work** in the space provided on this test.

Name: ______________________________

Question	Percent	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Question 1 (25 points)

a) Give the definition of the GG (Generalized Geography) problem.

b) Prove that $GG \in PSPACE$.
Be sure to include correctness and complexity bounds in your proof.

Question 2 (25 points)

a) Consider the following problem:

$$2COLOR = \{< G > \mid \begin{array}{l}\text{nodes of } G \text{ can be colored with 2 colors such that} \\ \text{no two nodes joined by an edge have the same color}\end{array} \}$$

b) Prove that $2COLOR \in P$.
Be sure to include correctness and complexity bounds in your proof.

Question 3 (25 points) Fill in the blanks with the following terms, where no term may be used **more than once** (any such occurrence will be marked WRONG).

a) ____________________ is NP-complete.

b) ____________________ is decided by a deterministic exponential time Turing machine.

c) ____________________ is PSPACE-complete.

d) ____________________ is decided by a deterministic polynomial time Turing machine.

e) ____________________ is decided by a deterministic polynomial space Turing machine.

- A_{LBA}
- $3SAT$
- $\overline{ALL_{NFA}}$
- $2SAT$
- $TQBF$

Question 4 (25 points)

a) Give the definition of the $3COLOR$ problem.

b) Prove that $3COLOR$ is a member of NP by constructing

   i) a polynomial time verifier for $3COLOR$, and

   ii) a polynomial time non-deterministic Turing machine that decides $3COLOR$.

c) Illustrate the polynomial time reduction $\neq SAT \leq_p 3COLOR$ for the boolean formula

$$\phi = (x \vee \overline{y} \vee z) \wedge (\overline{x} \vee y \vee \overline{z}) \wedge (\overline{x} \vee \overline{y} \vee \overline{z})$$

by constructing the corresponding graph G, and, if ϕ is satisfiable, indicating a satisfying truth assignment and the corresponding coloring for G.