CS 1511 Exam III

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15 April 2004

Instructions: This is a closed book, note and neighbor exam! You must show all work in the space provided on this test.

Name: _______________________

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Question 1 (25 points)

a) Give the definition of the $GG$ (Generalized Geography) problem.

b) Prove that $GG \in PSPACE$.
   Be sure to include correctness and complexity bounds in your proof.
Question 2 (25 points)

a) Consider the following problem:

\[ 2COLOR = \{ <G> \mid \text{nodes of } G \text{ can be colored with 2 colors such that no two nodes joined by an edge have the same color} \} \]

b) Prove that \( 2COLOR \in P \).
   
   Be sure to include correctness and complexity bounds in your proof.
**Question 3 (25 points)** Fill in the blanks with the following terms, where no term may be used *more than once* (any such occurrence will be marked WRONG).

a) \underline{3SAT} is NP-complete.

b) \underline{ALBA} is decided by a deterministic exponential time Turing machine.

c) \underline{ALLNFA} is PSPACE-complete.

d) \underline{2SAT} is decided by a deterministic polynomial time Turing machine.

e) \underline{TQBF} is decided by a deterministic polynomial space Turing machine.

- $A_{LBA}$
- $3SAT$
- $ALL_{NFA}$
- $2SAT$
- $TQBF$
Question 4 (25 points)

a) Give the definition of the 3COLOR problem.

b) Prove that 3COLOR is a member of NP by constructing
   
   i) a polynomial time verifier for 3COLOR, and
   
   ii) a polynomial time non-deterministic Turing machine that decides 3COLOR.

   
   c) Illustrate the polynomial time reduction \( \not= SAT \leq_p 3COLOR \) for the boolean formula

   \[
   \phi = (x \lor \overline{y} \lor z) \land (\overline{x} \lor y \lor \overline{z}) \land (\overline{x} \lor \overline{y} \lor \overline{z})
   \]

   by constructing the corresponding graph \( G \), and, if \( \phi \) is satisfiable, indicating a satisfying truth assignment and the corresponding coloring for \( G \).